Field Theory Analysis of Five Port Annular Microstrip Junction

W. Omer, D. Salem, A. Mohra and E. Hashish

I. Abstract

The five-port annular junction is solved using field theory analysis with the aid of finite elements method. Novel model for fringing fields is presented. Fields distributions and scattering matrices are obtained. Two five-port microstrip annular junctions are simulated using IE3D, realized and the measured S-parameters are found to be in good agreement with theoretical ones.

Keywords: *Five port, annular, finite element, fringing fields*

II. Introduction

In this paper the problem of symmetrical five port annular microstrip junction is presented. It is solved using field theory analysis with the finite elements method as a numerical tool. The waveguide model is used in which the boundary of the junction is assumed to be "magnetic wall"

Papers published in the annular microstrip junctions using the waveguide cavity model either neglected the fringing fields effect [1] or considered this effect with keeping the mean radius constant [2] and [3]. Both approaches yielded disagreeable results compared to experimental ones.

Due to the absence of a rigid model for calculating the fringing field effect of an annular resonator, it was the concern of this paper to present a semi-empirical model that does not keep the mean radius constant, but applies the effect of being an open structure on all the physical parameters of the circuit including the mean radius, which shifts the resonant frequency, to match with the experimental results.

Using this technique, the mode chart and the fields distributions in the junction, of different modes, are obtained. Following, both the impedance and the scattering matrices of the five port annular junction are obtained. Finally, two microstrip five port annular microstrip junctions are fabricated and their performances are measured. The experimental results are compared with the numerical ones. Very good agreement is achieved.

III. Field Theory Analysis

In microstrip junction, the separation between ground plane and the conductor, h, is arranged to be small with respect to the wave length in order to ensure that higher orders modes which vary in the Z-direction are suppressed [4]. This assumption implies that only the (E_z, H_x, H_y) field components exist.

Starting from Maxwell's equations and assuming that the fields vary as e^{jwt}, the wave equation is obtained as

$$(\nabla_t^2 \mathrm{E} + \mathrm{K}_{eff}{}^2)\mathrm{E}_Z = 0 \qquad (1)$$

Where $\nabla_t^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, E_Z is the longitudinal field component, and $\mathrm{K}_{eff}{}^2 = \omega^2 \varepsilon_o \varepsilon_r \mu_o \mu_r$. Equation (1) is known as the Helmholtz equation, it describes an eigen value problem, in which the values $K_o, K_1, K_2, \ldots etc.$ are the eigen values that correspond to resonant angular frequencies $\omega_o, \omega_1, \omega_2, \ldots$, etc.

The electric field on the contour of the junction is evaluated as the integration of the Green's function over the coupling ports. The Green's function is expanded as a series of complex eigenfunctions ϕ_a .

Consequently, the elements of the impedance matrix are defined as [5]:

$$Z_{ij} = \frac{jR_e k_{eff}}{w} \sum_{a=1}^{\infty} \frac{1}{k_a{}^2 - k_{eff}{}^2} \int_{p_i} \phi_a^*(r) \int_{p_j} \phi_a(r_o)\, \mathrm{dt_o} \qquad (2)$$

Where, R_e is the characteristic impedance of a microstrip line of width w and printed onto a substrate of a dielectric constant ε_r, and * denotes the complex conjugate. It is possible to evaluate ϕ_a analytically in a very small number of cases, and the most convenient method, in general, is to use a variational approach.

Miyoshi [6] introduced an approximate trial function ϕ_a', which causes the functional

$$F(\varphi_a'(r)) = \iint_s \left[|\nabla_t \varphi_a'(r)|^2 - k_a^2 |\varphi_a'(r)|^2 \right] ds - j\frac{k}{\mu}\oint_c (\varphi_a'(r)) \frac{\partial \varphi'(r)}{\partial n} dt \qquad (3)$$

to be minimized, satisfy both the differential equation and the boundary conditions for the eigen functions ϕ_a .The trial function ϕ_a' is an approximation to the exact function ϕ_a and it is expanded as

$$\phi_a'(r) = \sum_{i=1}^{M} c_i N_i \qquad (4)$$

Where c_i's denotes the complex expansion coefficients to be determined, N_i's are real basis functions, and M denotes the number of the basis function to be considered. These basis functions are chosen using *the finite element method.*

Substituting (4) into (3) and imposing the Rayleigh- Ritz condition

$$\frac{\partial F(\phi_a'(r))}{\partial c^*_i} = 0 \qquad (5)$$

Reduces the problem to a set of simultaneous homogenous equations of the form

$$[[A] - k^2_a [B]][C] = 0 \qquad (6)$$

$[A]$ and $[B]$ are symmetrical ($M \times M$) matrices, and C is ($M \times 1$) matrix [7]. Eq. (6) is known as the generalized eigen value problem. Solving Eq. (6) yields the eigen values k_a and the eigen vectors ϕ_a'.

For a matched five port annular junction, Fig.1, the equivalent admittance [5], is defined as

$$Y_{eq} = \frac{\sqrt{3}Y}{2\sin(\theta)} + j\left(\frac{1}{2\sin(\theta)} - 2\cot(\theta)\right) \qquad (7)$$

Where: θ is the electrical length and Y is the admittance level, which are the two parameters that may be adjusted to

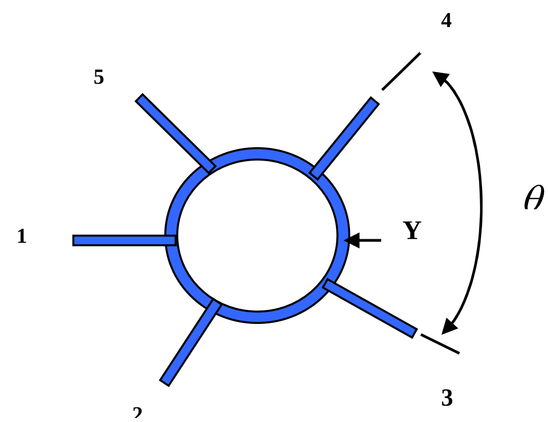


Fig.1. Schematic diagram of 5-port annular junction

obtain a match at a particular frequency. At the center frequency the imaginary and the real parts of Eq. (7) should be equal to zero and unity, respectively, to fulfill matching condition, which occurs at $\theta = 2\pi/\lambda$.

The technique presented is used to solve an annular resonator. Quadratic quadrilateral elements are used to discretize the resonator. The problem is first solved using a small number of elements and increases gradually until the results converge, at which the optimum number of elements is reached. The problem of a ring resonator of radius R=1m on a dielectric substrate of relative permittivity $\varepsilon_r = 2.2$ and dielectric thickness 0.7874 mm is solved. The first mode TM_{11} is studied for different number of elements for both narrow microstrip ring resonator ($W/R \to 0$) namely 0.05 and wide microstrip ring resonator ($W/R \to 1$) namely 0.9999. The results are compared to the analytical solution for the first mode [2]. For both problems the optimum results were obtained for 125-elements mesh with 425 nodes. The percentage error for the narrow width case $\approx 0.3\%$, whereas, for the wide width case $\approx 0.58\%$. Increasing the number of elements, more than 125 elements, slightly affected the percentage error. On the other hand, the computation time and memory requirements increased drastically.

One of the major benefits of using this technique is that the fields distributions within the junction is easily obtained, as an example the distributions of the lowest three order modes are shown in Fig. 2.

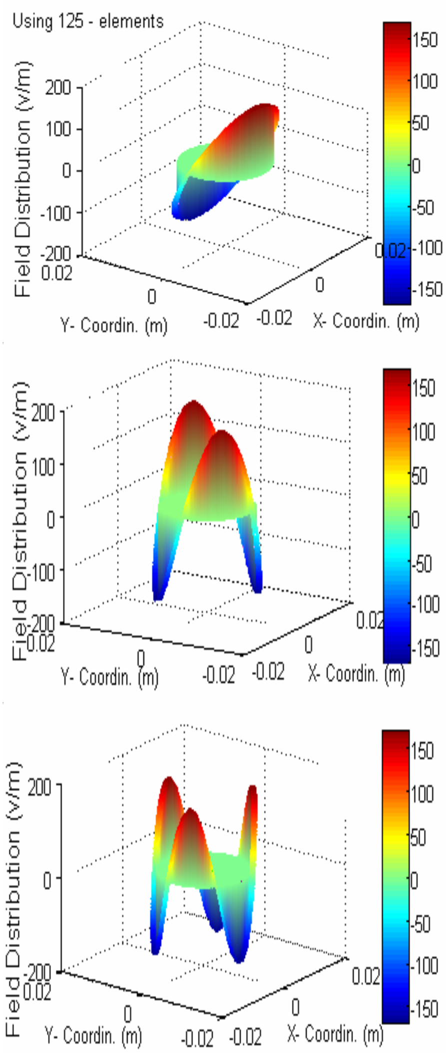


Fig. 2. Field distribution for the lowest three modes in annular junction resonator

Then, the mode chart for the microstrip annular resonator, for TM_{mno} modes is obtained, Fig. 3.

The Z-matrix of the five port annular junction is calculated using Eq. (2), from which the S- matrix is calculated. The number of modes included in the summation is about 30 modes.

The numerical results are obtained for a matched microstrip annular junction with mean *radius*, r_m, 9.90 mm and width, $w_{junction}$, 1.1361mm with feed line impedance, Z_o, $50\,\Omega$, substrate height, h, 0.635mm and relative permittivity, ε_r, 6.15, considering the boundary of the junction as a magnetic wall, except for the ports, Fig.4.

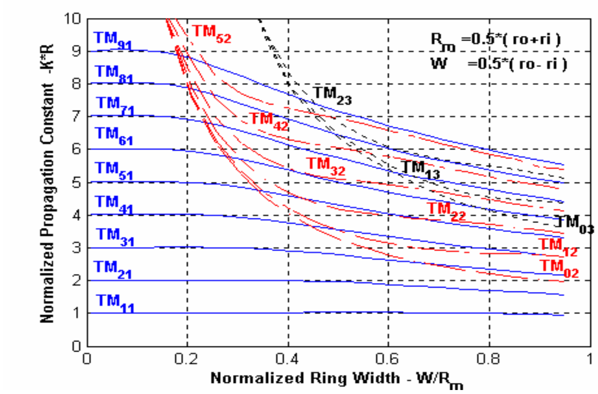


Fig. 3 Mode chart for microstrip annular resonator using finite element method

Fig.4 indicates that at the resonance frequency $f_0 \approx 2.0169\ GHz$, the input power is almost divided equally among the four ports (-6 dB). Next, the same problem was resolved using a model that caters for the fringing field effect. This model is highly dependent on the physical parameters of the problem, and is only applicable for the range $h = 0.635$ mm, $2.2 \le \varepsilon_r \le 15$, $2 \le f\ (\text{GHz}) \le 12$.

$$r_m = A + \frac{B}{f_{GHz}}$$

$$\Delta_{rm} = C + Dr_m + E \log r_m$$

$$r_m^* = r_m + \Delta_{rm}$$

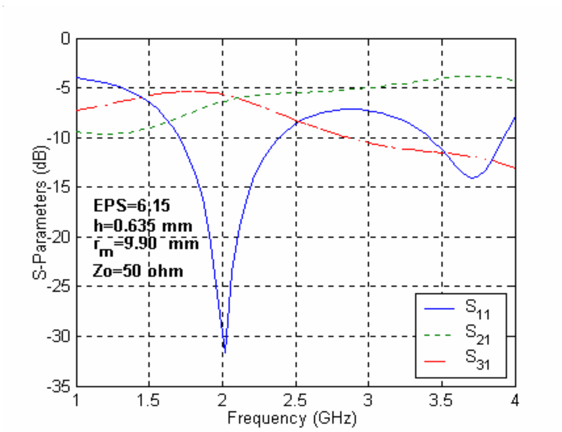


Fig.4. Variation of S-parameters with frequency with the magnetic wall assumption

Where:-

$$A = 0.5009 + 0.0022\varepsilon_r^{\,1.25} - 0.4889\log(\varepsilon_r)$$

$$B = 4.0086 + 48.4743\varepsilon^{-0.5} + 0.1045\cos(\varepsilon_r)$$

$$C = 0.0630 - 0.2407\varepsilon^{-1} - 0.0793\log(\varepsilon_r)$$

$$D = 0.2317 - 0.3910\varepsilon^{-1} + 0.0412\sin(\varepsilon_r)$$

$$E = 0.2156 \quad 0.5785\varepsilon^{-1}\tan(\varepsilon_r) + 0.4111\sin(\varepsilon_r)\cos(\varepsilon_r)$$

r_m : The physical mean radius of the five-port annular (mm).

r_m^* : The effective mean radius of the five-port annular junction (mm)

f_{GHz} : The center frequency (GHZ).

ε_r : The relative permittivity of the dielectric used

Δ_{rm} : The difference between the effective and physical values of mean radii

The numerical results using the above model are shown in Fig.5, where the center frequency is shifted to $\approx$ 2.42 GHZ.

V. Results

To check the validity of this technique two annular five port junctions were simulated using IE3D software package. Besides, they are fabricated using thin film technology in the microstrip laboratory and their characteristics are measured using the vector network analyzer.

The numerical, simulated and experimental results of the first

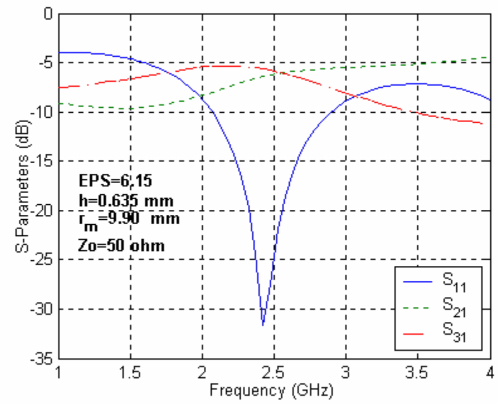
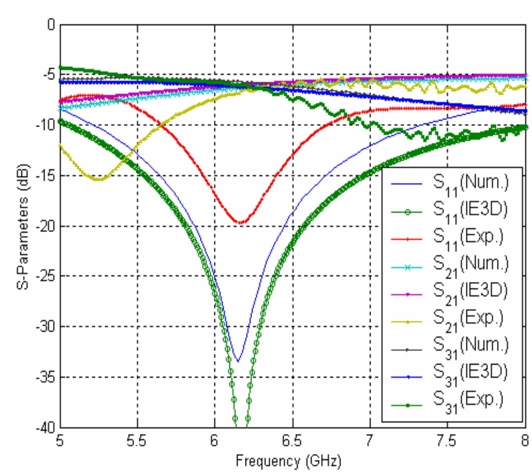


Fig.5. Variation of S-parameters with frequency using the model of the fringing effect

junction, $\varepsilon_r = 6.15$, $h = 0.635mm$, $r_m = 9.9$ mm and $Z_o = 50\Omega$, are shown in Fig. 6(a). The results of the second junction, $\varepsilon_r = 6.15$, $h = 0.635mm$, $r_m = 3.98$ mm and $Z_o = 50\Omega$, are shown in Fig. 6(b). The agreement of the three results is obvious. However, the results for larger mean radius are more satisfactory than those for smaller mean radius. This may be attributed to the fact that the coupling in the case of small mean radii is more considerable than the case of large mean radii, which is not taken into consideration in the theoretical solution.



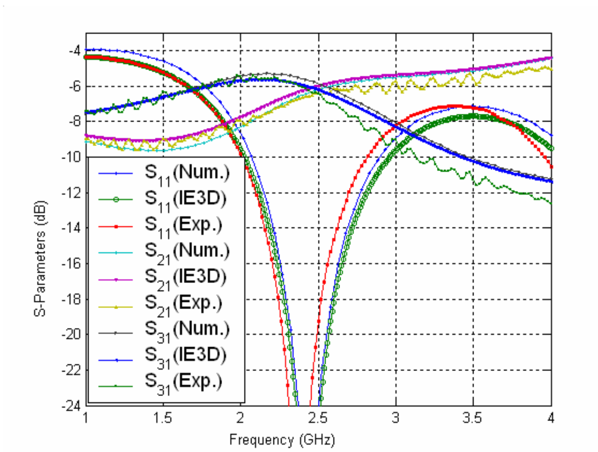


Fig. 6 The characteristics of the two annular five ports junction solved

(a) First Case:

$\varepsilon_r = 6.15$, $h = 0.635mm$,

$r_m = 9.9$ mm **and** $Z_o = 50\Omega$

(b) Second Case:

$\varepsilon_r = 6.15$, $h = 0.635mm$,

$r_m = 3.98$ mm

VI. CONCLUSION

This paper has described a finite element analysis of a symmetrical matched five-port microstrip annular junction. It was used to identify the mode chart of the resonator, some mode patterns and the S-parameters of the junction using a magnetic wall model. Besides, a new model that caters for the fringing field is presented.

Two five-port microstrip annular junctions were fabricated. The first has a center frequency of 2.42 GHz. whereas; the second has 6.15 GHz as its center frequency. The results of theoretical, simulated and measured results were compared and very good agreement is achieved.

VII. REFRENCES

1. T.S. Chu and Itoh, “Generalized Scattering matrix method for analysis of cascaded and offset microstrip step discontinuities,” IEEE Trans. Microwave Theory Tech., vol.MTT-34, pp.280-284, 1986.

2. Wu Y.S. and F.J. Rosenbaum, “Mode chart for microstrip resonators,” IEEE Trans. Microwave Theory and Tech. vol.MTT-21, No. 7, pp.487-489, July 1973.

3. R.P. Owens, “Curvature Effect in Microstrip ring resonators,” Electronics letters vol.12, no. 14, June 1976.

4. K. Chang, ***Microwave ring circuit and antenna***, Wiley Series in microwave and Optical Engineering, John Wiley and Sons, Inc., 1996.

5. E.R. Hansson and G.P. Ribblet, “An ideal six port network consisting of a matched reciprocal lossless five port and a perfect directional coupler,” IEEE Trans. Microwave Theory Tech., vol. MTT-31, pp.284-288, March 1993.

6. T. Miyoshi, S. Yamaguchi, and S. Goto, "Ferrite planner circuits in microwave integrated circuit,", IEEE Trans. Microwave Theory Tech., Vol. MTT-25, pp. 593-600, July 1977.

7. R. Lyon, J. Helszajn, "A finite element analysis of planar circulators using arbitrary shaped resonators", IEEE Trans. Microwave Theory Tech., MTT. 30, No. 11, pp.1964-1974, November 1982.